Published by Institute of Physics Publishing for SISSA/ISAS

Received: *December 15, 2003*
Accepted: *January 19, 2004*

On orbifold theory and $N = 2$, $D = 5$ gauged supergravity

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Abstract: We have studied the most general $N = 2$ supergravity in five dimensions in context with the orbifold theory based on $M_4 \times S^1/\mathbb{Z}_2$. Various ways to treat the supersymmetry with singular sources placed in orbifold fixed points were proposed in past. Supersymmetric branes were consistently introduced in a bulk where a gauged supergravity was present. In this paper we find that in the $N = 2, D = 5$ supergravity with general gauging, the possibility to obtain a supersymmetric brane world is constrained. Imposing the compatibility of supersymmetry transformation rules with the orbifold condition, we find the necessary and sufficient condition to obtain supersymmetric branes and bulk independently. We comment that the same condition guarantees naturally the presence of singular BPS solutions.

Keywords: Field Theories in Higher Dimensions, Discrete and Finite Symmetries, Supergravity Models.

http://jhep.sissa.it/archive/papers/jhep012004041/jhep012004041.pdf

JHEP01(2004)041

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1. Introduction

It's a very old idea that our four-dimensional world could be embedded in a higher dimensional space-time. Recently more attention concerned theories of large extra-dimensions, or brane-worlds, where the matter in our universe is confined to live on a $3+1$ dimensional hypersurface [1, 2]. Phenomenological interest of brane-worlds is twofold. First of all, they introduce extra-dimensions with a compactification scale testable by future experiments. Secondly, a more fundamental motivation of these models was to provide a solution to the hierarchy problem. In particular, in the Randall-Sundrum steps [2], a warped compactification has been explored. In such models, the bulk space-time is an AdS_5 or a slice of AdS_5 which induces on the branes an exponential decreasing warped factor.

Brane-world theories provide also alternative proposals for understanding important problems in cosmology. In particular, the dynamics of the bulk fields in a five dimensional model could be the origin of a cosmological constant or slowly rolling "quintessence" scalar fields which are principal current candidates of the dark energy. The smallness of the cosmological constant in warped brane-world cosmology is presented as result of a self-tuning. This idea advocates that, because of the warped factor, the would-be high vacuum energy on our brane is converted simply in the large curvature of extra-dimensions. This is the reason of the apparent vanishing 4-D cosmological constant. It's however known that the self-tuning domain wall suffers some kinds of instability and the presence of naked singularity in the bulk. Problems arising due to the singularities have been point out in [4, 5]. In some particular situations, singularity could be shielded by an additional source

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term, however this reintroduces a fine-tuning in the effective four dimensional theory [5]. Supersymmetry (SUSY) in the bulk provides a well known ingredient for preventing above problems. From the five dimensional supergravity point of view, self-tuning is nothing but requiring a BPS brane-world solution. This is one of the motivations for us to reconsider the BPS construction in a most general 5-D gauged supergravity with orbifold.

Brane-world arises naturally from gauged supergravities in five dimensions. In the literature, gauged supergravity theories in five dimensions were considered toward various directions, including or not localized interactions between bulk fields and four-dimensional gauge theories on branes [11]–[19]. To obtain warped brane-world solution, the bulk supergravity must be modified by an orbifold construction, for instance $S_1/\mathbb{Z}_2$, likes the original step of Randall-Sundrum scenarios. Other works consider also the supersymmetry breaking in warped space due to boundary condition [20]–[23]. Recently, a general consistency study of higher dimensional theories with symmetry breaking defects at the boundaries is given [23]. In this note we give a general framework of $N = 2$ $D = 5$ gauged supergravity in orbifold theory, completing some of the previous constructions [13, 14, 16]. We find that some interesting constraints on the bulk supergravity would occur to give back it consistent with orbifold condition.

From the five-dimensional point of view, brane-worlds are domain walls in space-time. Introducing SUSY in space-times with “fixed points” looks at first sight as a severe problem. In fact, in the orbifold theory, the Lorentz invariance in the direction orthogonal to the branes is broken. Then we expect that SUSY, being the square root of translations, is broken as well. The mechanism to restore the SUSY is to include on the branes the “pull-back” of suitable bulk fields in such a way that the supersymmetry variation of the branes compensates that of the bulk. This is the standard technique to incorporate the domain wall solution in supergravity theories. We shall not concentrate on the detail of these possibilities [11]–[19]. Our aim is to perform a general treatment on the consistency of the SUSY with the orbifold symmetry.

The note is organized as follows: in the next section 2, we present an introduction of smooth $N = 2$ $D = 5$ gauged supergravity, describing all possible supermultiplets. In section 3 we introduce the orbifold condition and his connection with the scalar manifold. In section 4 we will study in more detail the consistency between SUSY transformations and orbifold condition. We find some constraints on the scalar potential of the supergravity theory. In section 5 a general treatment of sigma model coupled to 5-D gravity is considered. The aim is to study the consistency of the brane action and the Israel junction conditions. In the end, in section 6, we present an explicit supersymmetrization of the general gauged supergravity with orbifold in absence of tensor supermultiplets .

2. $N = 2$, $D = 5$ gauged supergravity

The full couplings and gaugings of $D = 5$ $N = 2$ supergravity were described in [7, 8] and in the references therein. See also appendix B for some details of the full action helpful for our analysis. In this section, we shortly review the supermultiplets and scalar manifold present in this theory.

The most important fields of the supergravity theories are the components of the gravitational supermultiplet, the only supermultiplet present in pure supergravity [3]: $\{e^a_\mu, \psi^{\alpha i}_\mu, A_\mu\}$. They are the graviton e^a_μ, two gravitinos $\psi^{\alpha i}_\mu$ and a vector field A_μ (the graviphoton). To the pure supergravity, an arbitrary number of vector supermultiplets $\{A_\mu, \lambda^i, \phi\}$ can be coupled. The N=2 vector multiplet contains a vector field, an $\mathrm{SU}_R(2)$ doublet of spin-1/2 fermions λ^i and one real scalar field ϕ. One can introduce also matter supermultiplets: $\{\zeta^A, q^X\}$. A hypermultiplet contains a doublet of spin-1/2 fermions ζ^A and four real scalars q^X ($A = 1, 2$ $X = 1, \ldots, 4$). In detail, the supergravity theory with general matter coupling before the gauging is described by the following field content

$$\left\{ e_\mu{}^a, \psi^i_\mu, A^I_\mu, \lambda^{ix}, \zeta^A, \phi^x, q^u \right\}, \tag{2.1}$$

where an arbitrary number of vector supermultiplets (n_V) and hypermultiplets (n_H) is introduced. The index A now runs from 1 to $2n_H$. The scalars ϕ^x, q^u are coordinates of a scalar manifold $\mathcal{M}$ which is a direct product of a so called very special manifold and a quaternionic manifold:

$$\mathcal{M} = \mathcal{M}_V \otimes \mathcal{M}_H\,, \tag{2.2}$$

with $\phi^x \in \mathcal{M}_V$, $q^u \in \mathcal{M}_H$. $u = 1, \ldots, 4n_H$ and $x = 1, \ldots, n_V$ are the curved indexes labeling the coordinates of $\mathcal{M}$. In (2.1), λ^{xi} has two kinds of index, the components labeled by x transform as a vector under the tangent space group $SO(n_V)$ of vector scalar manifold $\mathcal{M}_V$ while i is an $\mathrm{SU}_R(2)$ rotation index. The index of ζ^A is flat.

The scalar manifold $\mathcal{M}$ admits in general a large group of isometries G and some of its subgroups K, as well as the $\mathrm{SU}_R(2)$ group, can be gauged using the vector fields present in the theory. One obtains in this way the so called gauged supergravity. An important property of the gauged supergravities is the presence of a potential for scalar fields. This is a necessary condition to the realization of AdS vacua and BPS solutions [9].

The non abelian gauging in a theory where only abelian vectors are present is problematic [7]. The reason is that if the gauge group K is non abelian, some spectator vector fields could be charged under K. Since before gauging all vectors are abelian, one has an inconsistency between the abelian gauge symmetry and the non abelian gauging. To solve this problem, the SUSY imposes that the K-charged spectator vectors have to be dualized to *self-dual* two-form fields. As a result, the $5D$ gauged supergravity may contain also tensor supermultiplets: $\{B_{\mu\nu}, \lambda^i, \phi\}$, here again an $\mathrm{SU}_R(2)$ doublet of spin-1/2 fermions and one real scalar field has been required. The tensor field satisfies the self-duality condition:

$$B_{\mu\nu} \propto \varepsilon_{\mu\nu\rho\sigma\tau} \partial^{[\rho} B^{\sigma\tau]} \tag{2.3}$$

to guarantees the same degree of freedom of a vector A_μ. The existence of tensor multiplets does not change the scalar content of the theory but the form of the scalar potential.

Now we review some important characteristics of the general $\mathrm{SU}_R(2)$ gauging. In the absence of hypermultiplets, the total global symmetry group factorizes into $\mathrm{SU}(2)_R \times G$, as a consequence of the $\mathrm{SU}(2)_R$-invariance of the scalar fields belonging to the vector multiplets. However, in general matter coupled supergravity theories, the R-symmetry group is non-trivially embedded into the quaternionic global symmetry group. In fact, in a quaternionic manifold, the holonomy group is $\mathrm{USp}(2n_H)$ times $\mathrm{USp}(2) \simeq \mathrm{SU}(2)$. The R-symmetry group is identified with the $\mathrm{USp}(2) \simeq \mathrm{SU}(2)$ contained in the holonomy group of the quaternionic manifold. Since $\mathrm{USp}(2)$ is evaluated on scalar manifold, when the whole $\mathrm{SU}(2)_R$ symmetry is gauged, a rotation of $\mathrm{SU}(2)_R$ may depend itself on scalar fields.

Five dimensions supergravity has a Chern-Simons term, a peculiar feature of odd-dimensional space-times. This term characterizes the "very special" manifold described by the scalars of vector multiplets. The gauging of the theory is then quite different from $4D$ gauged supergravity where the scalar manifold is special Kähler.

3. Scalar manifold and $\mathbb{Z}_2$

Now we can incorporate the supergravity theory just discussed in an orbifold construction: $M_5 = M_4 \otimes S_1/\mathbb{Z}_2$. Identifying the smooth circle S_1 with the fifth direction r, all fields of the theory are considered periodic in r. More precisely, if the radius of the circle is R, all fields obey $\Phi(r) = \Phi(r + 2\pi R)$. In fact alternative theories were studied in this regard to get spontaneous supersymmetry breaking. For instance, [20, 22] one can consider also the possibility to have a flipping in the boundary condition for fermionic fields, i.e. $\Psi(r) = -\Psi(r+2\pi R)$. We do not consider this possibility. The next step to get the orbifold theory on 5D gauged supergravity is to impose the $\mathbb{Z}_2$ symmetry which acts by $r \to -r$ on the fields. On the bosonic fields, the action of $\mathbb{Z}_2$ is the standard parity transformation on the fifth dimension:

$$\Phi(r) = \mathbf{P}(\Phi)\Phi(-r) = \pm\Phi(-r)\,.$$

The fermions $\psi_\mu^{\alpha i}, \lambda^i$, as well as the supersymmetry parameters ε^i, are symplectic (see appendix A) carrying an $\mathrm{SU}(2)_R$ index, their parity transformation needs to include an $\mathrm{SU}(2)_R$ rotation:

$$\lambda^i(r) = \mathbf{P}(\lambda)\gamma_5 M(q)^i{}_j\lambda^j(-r)\,, \tag{3.1}$$

$$\psi_\mu^i(r) = \mathbf{P}(\psi)\gamma_5 M(q)^i{}_j\psi^j(-r)\,, \tag{3.2}$$

$$\varepsilon^i(r) = \mathbf{P}(\varepsilon)\gamma_5 M^i(q)_j\varepsilon^j(-r)\,, \tag{3.3}$$

where

$$M^i{}_j = m_1(q)(\sigma_1)^i{}_j + m_2(q)(\sigma_2)^i{}_j + m_3(q)(\sigma_3)^i{}_j\,, \tag{3.4}$$

with m_1, m_2, $m_3 \in$ real functions of q[1] and $\mathbf{P}(\Psi) = \pm$, $\Psi \equiv (\lambda, \psi, \varepsilon)$. Due to the parity property of M: $M^2 = 1$, the coefficients m_i must form a vector with unit norm. The news here with respect to the previous works is that we introduce a dependence of the rotation coefficients on the scalar fields. The reason is that, in the general gauged 5D supergravity under consideration, the whole $\mathrm{SU}(2)_R$ group has been gauged rather than its abelian subgroup $\mathrm{U}_R(1)$. On the other side, as already discussed in the previous section, $\mathrm{SU}(2)_R$ is just identified with the subgroup $\mathrm{USp}(2)$ of the holonomy group of the quaternionic manifold. As will also be shown explicitly in the following, m_i depends in fact only on scalars q.

[1] In the case of minimal gauged supergravities with an abelian gauging in a specific direction, the orbifold rotation $M^i{}_j$ must be constant.

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Only the even field will survive on fixed points of the orbifold where we will put the branes. What we are going to do in the next section is to assign the correct parity properties to the fields in order to keep the bulk action even and all supersymmetric transformations consistent. This is a fundamental step also for a successful supersymmetrization of the branes themselves. As a first and obvious requirement, we shall assign a negative parity to the r coordinate: $\mathbf{P}(\partial_5) = -1$. Furthermore, $\mathbf{P}({e_m}^{\tilde{a}}) = 1$ because the brane action for the 4-dimensional hypersurface is multiplied by the determinant of the vierbein ${e_m}^{\tilde{a}}$.

4. Susy transformations and $\mathbb{Z}_2$

For clearness, we recall the full SUSY transformation rules for the general gauged $N = 2$ $D = 5$ supergravity (for the notation, see appendix B):

$$
\begin{aligned}
\delta_\varepsilon e^a_\mu &= \frac{1}{2}\bar{\varepsilon}^i\gamma^a\psi_{\mu i}\,, && (4.1)\\
\delta_\varepsilon\psi_{\mu i} &= D(\widehat{\omega})\varepsilon_i + \frac{i}{4\sqrt{6}}h_I e^a_\mu\left(\gamma_{abc}\varepsilon_i - 4\eta_{ab}\gamma_c\varepsilon_i\right)\widehat{H}^{bc\,I} - \delta_\varepsilon q^X\omega_{X\,i}{}^j\psi_{\mu j} -\\
&\quad - \frac{1}{12}e^a_\mu\gamma_{ab}\varepsilon^j\,\overline{\lambda}^x_i\gamma^b\lambda^x_j + \frac{1}{48}e^a_\mu\gamma_{abc}\varepsilon^j\,\overline{\lambda}^x_i\gamma^{bc}\lambda^x_j + \frac{1}{6}e^a_\mu\varepsilon^j\,\overline{\lambda}^x_i\gamma_a\lambda^x_j -\\
&\quad - \frac{1}{12}e^a_\mu\gamma^b\varepsilon^j\,\overline{\lambda}^x_i\gamma_{ab}\lambda^x_j + \frac{1}{8}e^a_\mu\gamma^{bc}\varepsilon_i\,\overline{\zeta}_A\gamma_{abc}\zeta^A + \frac{i}{\sqrt{6}}g_R\,e^a_\mu\gamma_a\varepsilon^j P_{ij}\,, && (4.2)\\
\delta_\varepsilon\phi^x &= \frac{i}{2}\bar{\varepsilon}^i\lambda^x_i && (4.3)\\
\delta_\varepsilon A^I_\mu &= -\frac{1}{2}e^a_\mu\,\bar{\varepsilon}^i\gamma_a\lambda^x_i\,h^I_x + \frac{i\sqrt{6}}{4}\overline{\psi}^i_\mu\varepsilon_i\,h^I\,, && (4.4)\\
\delta_\varepsilon\lambda^x_i &= -\frac{i}{2}\gamma^a\varepsilon_i\,\widehat{D}_a\phi^x - \delta_\varepsilon\phi^y\Omega_y{}^{xz}\lambda^z_i - \delta_\varepsilon q^X\omega_{X\,i}{}^j\lambda^x_j + \frac{1}{4}h^x_I\gamma^{ab}\varepsilon_i\,\widehat{H}^I_{ab} -\\
&\quad - \frac{i}{4\sqrt{6}}T^x{}_{yz}\left[-3\varepsilon^j\,\overline{\lambda}^y_i\lambda^z_j + \gamma_a\varepsilon^j\,\overline{\lambda}^y_i\gamma^a\lambda^z_j + \frac{1}{2}\gamma_{ab}\varepsilon^j\,\overline{\lambda}^y_i\gamma^{ab}\lambda^z_j\right] +\\
&\quad + g_R\varepsilon^j P^x_{ij} + gW^x\varepsilon_i\,,\\
\delta_\varepsilon B^M_{\mu\nu} &= 2D_{[\mu}\delta_\varepsilon A^M_{\nu]} + \frac{i}{8}g e^b_\mu e^a_\nu\,\bar{\varepsilon}^i\gamma_{ab}\lambda^x_i\,h_N\Omega^{MN} + g\frac{\sqrt{6}}{8}e^a_\mu\,\overline{\psi}^i_\nu\gamma_a\varepsilon_i\,\Omega^{MN}h_N\,, && (4.5)\\
\delta_\varepsilon q^X &= -i\bar{\varepsilon}^i\zeta^A\,f^X_{iA}\,, && (4.6)\\
\delta_\varepsilon\zeta^A &= -\frac{i}{2}\gamma^a\varepsilon^i\widehat{D}_a q^X\,f^A_{iX} - \delta_\varepsilon q^X\omega_{X\,B}{}^A\zeta^B + g\varepsilon^i\mathcal{N}^A_i\,. && (4.7)
\end{aligned}
$$

We begin to choose the components of the gravitino in the directions transverse to '5', ψ_m, to be even. We will return to this point when we study the supersymmetry on the branes. First let us concentrate on the ungauged part of the SUSY transformation. Only the following parity assignments are then consistent:

$$
\begin{aligned}
\mathbf{P}({e_m}^{\tilde{a}}) &= \mathbf{P}({e_5}^5) = 1\,, & \mathbf{P}(A^I_5) &= 1\,,\\
\mathbf{P}(q) &= \mathbf{P}(\phi) = 1\,, & \mathbf{P}(\psi_m) &= \mathbf{P}(\epsilon) = 1\,,\\
\mathbf{P}({e_5}^{\tilde{a}}) &= \mathbf{P}({e_m}^5) = -1\,, & \mathbf{P}(A^I_m) &= \mathbf{P}(\Lambda^I) = -1\,,\\
\mathbf{P}(\psi_5) &= \mathbf{P}(\lambda) = \mathbf{P}(\zeta) = -1\,. & & \qquad (4.8)
\end{aligned}
$$

What is left over are the parity properties of the tensor fields which must be considered apart. As we have already seen, the tensor fields in $N = 2$ $D = 5$ gauged supergravity are self-dual:

$$B^M_{\mu\nu} = \frac{1}{m_T}\varepsilon_{\mu\nu\rho\sigma\tau}\partial^{[\rho}B^{M\sigma\tau]}\,, \tag{4.9}$$

where $M = 1, \ldots, n_T$ labels the tensor multiplets and m_T has dimension of a mass. A naive statement from the SUSY transformation (4.5) provides the choice

$$\mathbf{P}(B_{m5}) = -\mathbf{P}(B_{mn}) = 1\,.$$

However it is easy to point out that this choice is not consistent with the condition (4.9). To show it, we can consider the component mn of (4.9)

$$B_{mn} = \frac{1}{m_T}\left(\varepsilon_{mn5pq}\partial^{[5}B^{pq]} + \varepsilon_{mnl5q}\partial^{[l}B^{5q]} + \varepsilon_{mnlp5}\partial^{[l}B^{p5]}\right). \tag{4.10}$$

Self-duality condition (4.10) mixes the B_{m5} components with the B_{mn} ones. Since $\mathbf{P}(\partial^l) = 1$ and $\mathbf{P}(\partial^5) = -1$, (4.10) alone cannot be realized with defined parity assignment of $B_{\mu\nu}$. The only way to get around this problem is to introduce parity property also for the mass of tensors m_T:

$$m_T \longrightarrow m_T\varepsilon(r)\,,$$

where $\varepsilon(r)$ is the step function. We then conclude that *tensor fields are consistent with the orbifold theory only if they have an odd mass.*

It remains to analyze the consistency of the SUSY transformation rules containing the gauge constant g_R. The two relevant terms in this case appear in the trasformation of the gravitino and of λ^x_i:

$$\frac{i}{\sqrt{6}}g_R\, e^a_\mu\gamma_a\varepsilon^j P_{ij}\,, \qquad g_R\varepsilon^j P^x_{ij}\,. \tag{4.11}$$

Taking the parity of both sides, we obtain the following two conditions:

$$M^j{}_i P^k{}_j\epsilon_k = -\mathbf{P}(g_R)P^j{}_i M^k{}_j\epsilon_k\,, \tag{4.12}$$
$$M^j{}_i P^{xk}{}_j\epsilon_k = -\mathbf{P}(g_R)P^{xj}{}_i M^k{}_j\epsilon_k\,, \tag{4.13}$$

where we allowed a possible flipping of the sign in g_R when it goes through the origin of the fifth dimension. From the property of very special geometry ((B.3) in appendix B), we have the relation $P^x_{ij} = -\sqrt{\frac{3}{2}}P^{;x}_{ij}$ where $^{;x}$ denotes ∂_{ϕ^x}. Then confronting the derivative of (4.12) with (4.13), the $\mathrm{SU}(2)_R$ projector M can be writing in the following form:

$$M_{ij} = f(\Phi)M^0_{ij}$$

with M^0_{ij} independent on ϕ^x and f a scalar function with $f^2 = 1$. Note that it is not restrictive to impose $f \equiv 1$. In this way M does not depend anymore on ϕ^x.

The prepotential P_{ij}, as well as M_{ij}, is often written in the base of the Pauli matrices σ^s in the following:

$$P_{ij} = i(P^s\sigma^s)_{ij}\,, \qquad M_{ij} = i(M^s\sigma^s)_{ij}\,.$$

The SUSY transformation laws are $\mathbb{Z}_2$ invariant if we may decompose

$$g_R \bar{P} = g_R^{(1)} \varepsilon(r) \bar{P}^{||} + g_R^{(2)} \bar{P}^{\perp} \,, \tag{4.14}$$

where $\bar{P}^{||}$ is parallel to $\bar{M}$ while $\bar{P}^{\perp}$ orthogonal to $\bar{M}$. Or equivalently, $P_{ij}^{||}$ commutes with M_{ij} and $P_{ij}^{\perp}$ anticommutes with M_{ij}. In the former case, the gauge coupling is proportional to the step function $\varepsilon(r)$.

The different features for the parallel and the orthogonal components are well known [14, 20]. The presence of $\bar{P}^{\perp}$ breaks the BPS domain wall solutions. The fact is one would not solve Killing equations everywhere in the bulk because odd fields are discontinuous in fixed points of the orbifold. To restore the SUSY, the techniques used are to add new terms proportional to δ, delta of Dirac, in SUSY transformation laws and consequently a brane action to guarantee the closure of $N = 2$ supersymmetry. However in [14] it has been pointed out that this mechanism does work only in the absence of hypermultiplets. The main reason is that in presence of a general scalar manifold, the equations of motion of the scalar fields destroy the desired closure of the SUSY algebra. From the previous discussion, since this note considers general matter coupled supergravity, we renounce to the possibility to have an orthogonal component in (4.14) therein.

Now let us return to the consistency conditions (4.12) and (4.13). Since we consider only the parallel component of the prepotential, P_{ij} must be equal to M_{ij} times a scalar function:

$$P_{ij} = A(q,\phi) M_{ij}(q) \,. \tag{4.15}$$

In other words, we can decompose the prepotential P_{ij} in its norm $W(q,\phi)$, which is precisely the superpotential, multiplied by its $\mathrm{SU}_R(2)$ phase Q^s:

$$P^s = W(q,\phi) Q^s \,, \qquad \text{with the constraint} \qquad \partial_\phi Q^s = 0 \,. \tag{4.16}$$

The most important consequence of this result (4.16) is, in agreement with [9], that the scalar potential $V(q,\phi)$ can then be written in a form that has been put forward for gravitational stability neglecting the tensor contribution:

$$V = -6W^2 + \frac{9}{2} g^{\Lambda\Sigma} \partial_\Lambda W \partial_\Sigma W \,. \tag{4.17}$$

$g_{\Lambda\Sigma}$ represents the metric of the complete scalar manifold. $N = 2$, $D = 5$ gauged supergravities with scalar potential given in the stable form had already been analyzed to establish supersymmetric domain walls flow in these theories [10, 9]. In particular, in [9], the authors impose the condition (4.16) by hand in order to obtain stable form of the scalar potential and consequently analyze supersymmetric domain wall solutions. Our result, on the contrary, points out that:

> *the consistency between the orbifold conditions and the SUSY transformations yields automatically the stable form of the scalar potential (4.17).*

Furthermore, in the following sections we will show that the stable form of the scalar potential is also necessary to successful supersymmetrization of the full brane-bulk system.

5. Brane action

The gauge coupling multiplied by a step function $g_R\varepsilon(r)$ introduces a singularity in the SUSY transformations law. The bulk supergravity will not longer be supersymmetric and its variation must be proportional to the delta function centered on the fixed points of the orbifold. It's then necessary to introduce a brane action to cancel this variation. In this section, we will show how the stable form imposed on the superpotential is required to obtain a consistent brane action.[2] The brane action is also accountable to the matching of various slides of the bulk supergravity in the orbifold background. To understand all these points, let us concentrate on the scalar part of the bulk action:

$$\begin{aligned} e^{-1}\mathcal{L}_{scalar} &= \frac{1}{2}R - \frac{1}{2}g_{XY}\partial_\mu q^X \partial^\mu q^Y - \frac{1}{2}g_{xy}\partial_\mu\phi^x\partial^\mu\phi^y - V(\phi,q) \\ &= \frac{1}{2}R - \frac{1}{2}g_{\Lambda\Sigma}\partial_\mu\Phi^\Lambda\partial^\mu\Phi^\Sigma - V(\phi,q)\,, \end{aligned} \tag{5.1}$$

where $V(\Phi)$ is the scalar potential expressed in his stable form (4.17). Remember that the metric $g_{\Lambda\Sigma}$ represents the complete scalar manifold and we assume the tensor contributions to the scalar potential to vanish.

The theory described by (5.1) can be seen as a Sigma model coupled to gravity which is a generalization of scalar-tensor gravity. Domain wall solutions in scalar-gravity are quite interesting issues [6, 10]. Here we will examine the Einstein theory in the Sigma model case, because the metric $g^{\Lambda\Sigma}$ in (5.1) depends itself on the scalar fields.

We look for static solutions with a metric ansatz which depends only on the transverse direction of the brane and is minkowskian flat on the brane.

$$ds^2 = e^{-A(r)}dx_4^2 + dr^2\,. \tag{5.2}$$

The equations of motion which follow from the action (5.1) with the ansatz (5.2) are:

$$\frac{3}{2}A''(r) = g_{\Lambda\Sigma}\Phi'^\Lambda(r)\Phi'^\Sigma(r), \tag{5.3}$$

$$\frac{3}{2}A'(r)^2 = \frac{1}{2}g_{\Lambda\Sigma}\Phi'^\Lambda(r)\Phi'^\Sigma(r) - V[\Phi(r)], \tag{5.4}$$

$$\left(g_{\Lambda\Sigma}\Phi'^\Sigma(r)\right)' - A'(r)g_{\Lambda\Sigma}\Phi'^\Sigma(r) = 2\frac{\partial V[\Phi]}{\partial\Phi}\,, \tag{5.5}$$

where the primes denote derivatives with respect to r. Since the potential $V[\Phi]$ is given in terms of the superpotential W in the stable form, it is very easy to see the general solution of the system (5.3)–(5.5):

$$\Phi'^\Lambda(r) = 3g^{\Lambda\Sigma}\frac{\partial W[\Phi(r)]}{\partial\Phi^\Sigma(r)}\,, \tag{5.6}$$

$$A'(y) = 2W[\Phi(r)]\,. \tag{5.7}$$

[2]Tensor multiplets give otherwise contribution to the scalar potential which destroys the stable form and we will not consider them here.

We can introduce now a general brane action at a fixed point $r = r_0$

$$\int d^5x \sqrt{-g}\, f[\Phi]\, \delta(r - r_0)\,. \tag{5.8}$$

The equations of the motion will be modified and in order to construct continuous solutions in the bulk with now two slides separated by the brane, they need to respect a set of supplementary boundary conditions at the brane:

$$\Delta\Phi'^{\Lambda}(r) \;=\; 3\Delta g^{\Lambda\Sigma}\frac{\partial W[\Phi(r)]}{\partial\Phi^{\Sigma}(r)} = g^{\Lambda\Sigma}\frac{\partial f[\Phi(r)]}{\partial\Phi^{\Sigma}(r)}\Big|_{r=r_0}, \tag{5.9}$$

$$\Delta A' \;=\; 3\Delta W[\Phi(r)] = f[\Phi(r)]\Big|_{r=r_0}, \tag{5.10}$$

where ΔF indicates the jump of a discontinuous function F at $r = r_0$, $F(r_0^+) - F(r_0^-)$. These conditions are known as Israel junction conditions. Obviously $\phi(r)$ and $A(r)$ are automatically continuous across the boundary in the orbifold background. From (5.9) and (5.10), it is easy to check that in order to be consistent with the boundary condition for any r_0, the coefficient $f[\Phi]$ in front of the determinant of the 4-dimensional metric in (5.8) must be exactly three times the superpotential W. Since our background is $\mathbf{S^1}/\mathbb{Z}_2$, there are 2 fixed points at $r = 0$ and $r = \pi R$, the brane action is

$$S_{\text{brane}} \supset -\int d^5x\, (\delta(r) - \delta(r - \pi R))\, e_{(4)} 3W\,. \tag{5.11}$$

where $e_{(4)}$ is the determinant of the vierbein $\epsilon_m^{\tilde{a}}$. Now the importance of the stable form of the potential in the orbifold theory appears clear. In the absence of the condition obtained in (4.16), it's impossible to introduce a brane consistent with the boundary condition for any radius R of the extra-dimension. From the supergravity point of view, this conclusion is mainly due to the fact that junction conditions are also conditions for the existence of BPS brane-world solutions [10].

6. Supersymmetrize brane and bulk

In this section, we extend the method explained in [14] to supersymmetrize 5-dimensional supergravity coupled with vector multiplets and hypermultiplets with general gauging. As a result, we find that the assignment of the parity of various fields is fundamental in order to supersymmetrize branes independently to bulk supergravity. The full action is described in appendix B. The method consists in replacing the odd gauge coupling constant $g_R\varepsilon(r)$ by a new smooth scalar field $G(r)$ which is invariant under SUSY transformations. One then introduces a four form $A_{\mu\nu\rho\sigma}$ as a lagrangian multiplier to impose $G(r)$ to be everywhere constant on shell:

$$S_A = \frac{1}{4!}\int d^5x \varepsilon^{\mu\nu\rho\sigma\tau} A_{\mu\nu\rho\sigma}\partial_\tau G\,. \tag{6.1}$$

Varying the action with respect to $A_{\mu\nu\rho\sigma}$ we obtain the usual supergravity action in the bulk. The advantage of this formulation arises when we wish to supersymmetrize the branes and the bulk independently. The replacement $g \to G$ breaks the original $5D$ SUSY, but the SUSY variation of (6.1) may restore it.

$$\begin{aligned} e^{-1}\delta(\epsilon)\mathcal{L}_{5d} &= \left[-\overline{\psi}^i_\mu\gamma^{\mu\nu\rho}\epsilon^j A^I_\nu P_{Iij} - \frac{i\sqrt{6}}{8}\overline{\psi}^i_\mu\gamma^{\mu\nu}\epsilon^j P_{ij} + 2\overline{\lambda}^i_x\gamma^\nu\epsilon^j P^x_{ij} - 2\overline{\varepsilon^i}\gamma^\nu\zeta^A\mathcal{N}_{iA}\right]\partial_\nu G \\ &= B^\nu\partial_\nu G\,. \end{aligned} \tag{6.2}$$

This variation is proportional to $\partial_\nu G$ since when g_R is constant, the gauged action is supersymmetric. If we define the SUSY transformation of $A_{\mu\nu\rho\sigma}$ as

$$\delta(\epsilon)\frac{1}{4!}\varepsilon^{\mu\nu\rho\sigma\tau}A_{\nu\rho\sigma\tau} = -B^\mu\,, \tag{6.3}$$

the whole bulk remains supersymmetric and the SUSY algebra will be realized on-shell.

The boundary action consists of two parts. The first one is (5.11), already considered in the previous section. The second part is the induced action (6.1) on the fixed four-dimensional sub-spaces of the orbifold. In summary, we have

$$S_{brane} = -2G(r)\int d^5x\;(\delta(r)-\delta(r-2\pi r_0))\left(e_{(4)}3W + \frac{1}{4!}\varepsilon^{mnpq}A_{mnpq}\right). \tag{6.4}$$

Let's observe that the second part of the brane action (6.4) is consistent with the parity assignment of G:

$$\mathbf{P}(G) = -1 \qquad\longrightarrow\qquad \mathbf{P}(A_{mnpq}) = +1\,.$$

Having introduced the brane action, the field equation of $G(r)$ becomes:

$$\partial_5 G(r) = 2g_R\,(\delta(r)-\delta(r-2\pi r_0))\,, \tag{6.5}$$

which implies $G(r) = g_R\varepsilon(r)$. To check the SUSY invariance of the brane action we use the formula

$$\delta_\varepsilon W = W_{,x}\delta_\varepsilon\phi^x + W_{,u}\delta_\varepsilon q^u$$

and the variation of A_{mnpq} from (6.2).

$$\begin{aligned} \delta_\varepsilon S_{\rm brane} &= -3g\int d^5x\,(\delta(r)-\delta(r-2\pi r_0))\times \\ &\times\Big[W e_{(4)}\overline{\epsilon}^i\gamma^{\tilde a}e^m_{\tilde a}\left(\psi_{mi} - i\gamma_5 Q_{ij}\psi^j_m\right) + W_{,x}\overline{\epsilon}^i i\left(\lambda^x_i + i\gamma_5 Q_{ij}\lambda^{xj}\right) + \\ &\quad + \overline{\epsilon}^i\zeta^A\left(-2iW_{,u}f_{iuA} - \frac{4}{3}\gamma_5\mathcal{N}_{iA}\right)\Big]. \end{aligned} \tag{6.6}$$

where Q_{ij} will be replaced by M_{ij}, see (4.15) and (4.16). The terms in the second line of (6.6) come from the vector multiplets and are result has been obtained in [14]. The only difference, in our case, is that Q_{ij} now depends on scalars q^u of the quaternionic manifold. The terms in the third line of (6.7) are the contribution of hypermultiplets. For the parity assignment (4.8), the following fields or combination of fields

$$\psi^-_{mi} = \psi_{mi} - i\gamma_5 M_{ij}\psi^j_m\,, \qquad (\lambda^x_i)^- = \lambda^x_i + i\gamma_5 M_{ij}\lambda^{xj}\,, \qquad \zeta^A\,, \tag{6.7}$$

have parity odd and then they vanishes on the branes. In principle, there is a second possibility of parity assignment changing simultaneously the parity of the SUSY transformation parameter ϵ and of ψ_μ^i. In theories where only vector multiplets are present, the second choice is equivalent of the first one because we can add an additional minus sign when we identify Q_{ij} with M_{ij}. But when we consider also the hypermultiplets, a negative assignment for ϵ implies a positive one for ζ^A and the variation (6.6) does not vanish anymore. We check that in order to obtaining supersymmetric branes, (4.8) is the only possible assignment of parity.

7. Conclusion

In summary, this note reviews some salient aspects concerning the combination of gauged supergravity theories with recent well promising orbifold theories. $5D$ supergravity realization of phenomenologically testable brane-world models is a very attractive issue. Many mechanisms were introduced in order to implement supersymmetry in space-time with singular sources. Although in the last few years, solutions in this regard were obtained in different way, it still remains an interesting stage to explore some general connection between $5D$ gauged supergravity and orbifold theory. Motivated by this intent, our goal has focused on the auto-consistency of the SUSY algebra, when matter fields are present in the bulk, with the orbifold construction and with the brane-world solution. An important requirement in searching singular BPS solutions in this theory is the stable form of the scalar potential. In fact, in general matter coupled supergravity, a gravitational stable potential is not automatic. In a different way, we find that orbifold theory can guarantee a stable form of scalar potential by the orbifold projection. As a result, the existence of SUSY brane-world solutions can emerge naturally. Furthermore, these solutions are important candidates also for cosmological scalar-tensor gravity in context of a more fundamental background. In this perspective, it's interesting to find a specific model where hypermultiplets lead to a supersymmetric brane-world solution with warped compactification and we hope to come back to this point in near future.

Acknowledgments

We are glad to thank Mario Tonin for helpful conversations and for pointing out some misprints. We are particularly grateful to Maria Laura Alciati and Luca Scarabello for their interest and encouragement in our work. This work is partially supported by the EU under TMR contract HPRN-CT-2000-00148.

A. Notations

The metric is $(-++++)$, and $[ab]$ denotes antisymmetrization $\frac{[ab-ba]}{2}$. We use the following indexes

$$
\begin{array}{lll}
\mu & 0,\ldots,3,5 & \text{local spacetime} \\
m & 0,\ldots,3 & \text{4D local spacetime} \\
a & 0,\ldots,3,5 & \text{tangent spacetime} \\
\tilde{a} & 0,\ldots,3 & \text{tangent spacetime in 4 dimensions}\,.
\end{array} \tag{A.1}
$$

The Levi–Civita tensor satisfies

$$
\varepsilon_{abcde}\varepsilon^{abcde} = -5!\,, \qquad \varepsilon^{\mu\nu\rho\sigma\tau} = e e^{\mu}_{a} e^{\nu}_{b} \cdots e^{\tau}_{e} \varepsilon^{abcde}\,. \tag{A.2}
$$

The five–dimensional $(\gamma^a)_\alpha{}^\beta$ matrices satisfy the Dirac algebra

$$
\{\gamma^a, \gamma^b\} = 2\,\eta^{ab}\,.
$$

$\gamma^{a_1\ldots a_n}$ means the antisymmetrized product of n γ matrices with weight one: $\gamma^{[a_1}\ldots\gamma^{a_n]}$.

Symplectic-Majorana spinors carrying the USp(2) doublet index $i=1,2$ which is raised and lowered with the invariant USp(2) tensor $\epsilon_{12}=\epsilon^{12}=1$, in NW-SE convention:

$$
X^i = \varepsilon^{ij} X_j\,, \qquad X_i = X^j \varepsilon_{ji}\,. \tag{A.3}
$$

The symplectic–Majorana condition on a generic spinor $\lambda_{\alpha i}$ reads

$$
\overline{\lambda}^i \equiv \lambda_i^\dagger \gamma^0 = \left(\lambda^i\right)^T C\,, \tag{A.4}
$$

where $\overline{\lambda}$ is the usual Dirac conjugate and C is the charge conjugation matrix satisfying $C^T = -C = C^{-1}$.

B. $N=2$ $D=5$ gauged supergravity

$$
\begin{aligned}
e^{-1}\mathcal{L} = & -\frac{1}{2}R(\omega) - \frac{1}{2}\bar{\psi}^i_\mu \gamma^{\mu\nu\rho}\left(\nabla_\nu(\omega)\psi_{\rho i} + \mathcal{D}q^u \omega_{uij}\psi^j_\rho + g_R A^I_\nu P_{Iij}\psi^j_\rho\right) - \\
& - \frac{1}{2}\bar{\lambda}^i_x \gamma^\mu [\nabla_\mu(\omega)\lambda^x_i + g_R A^I L_I{}^{xy}(\phi)\lambda_{yi} + \\
& \qquad + \Gamma^x_{yz}(\mathcal{D}_\mu\phi^y)\lambda^z_i + \mathcal{D}_\mu q^u \omega_{ui}{}^j \lambda^x_j + g_R A^I_\mu P_{Iij}\lambda^{xj}] - \\
& - \bar{\zeta}_A \gamma^\mu \nabla_\mu(\omega)\zeta^A + \bar{\zeta}_A\gamma^\mu\left(\mathcal{D}_\mu q^u \Delta_{uB}{}^A + g_R A^I_\mu \omega_{IB}{}^A\right)\zeta^B + \\
& + \frac{e^{-1}}{6\sqrt{6}} C_{IJK}\varepsilon^{\mu\nu\rho\sigma\lambda}\Big\{F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda + \frac{3}{2}gF^I_{\mu\nu}A^J_\rho(f^K_{LF}A^L_\sigma A^F_\lambda) + \\
& \qquad + \frac{3}{5}g^2(f^J_{GH}A^G_\nu A^H_\rho)(f^K_{LF}A^L_\sigma A^F_\lambda)A^I_\mu\Big\} - \frac{1}{4}a_{IJ}\mathcal{F}^I_{\mu\nu}\mathcal{F}^{J\mu\nu} + \\
& + g_R\left[\frac{i}{2\sqrt{6}}\,\overline{\lambda}^{ix}\lambda^{jy}\,P^{xy}_{ij} - \overline{\lambda}^{ix}\gamma_\mu\psi^{j\mu}\,P^x_{ij} - \frac{i\sqrt{6}}{8}\,\overline{\psi}^{i\mu}\gamma_{\mu\nu}\psi^{j\nu}\,P_{ij}\right] + \\
& + g\left[2\,\overline{\psi}^{i\mu}\gamma_\mu\zeta^A\,\mathcal{N}_{iA} - 2i\,\overline{\zeta}^A\lambda^{xi}\,\mathcal{M}_{Aix} + \overline{\zeta}^A\zeta^B\,\mathcal{M}_{AB}\right] - \\
& - \frac{1}{2}g_{\Sigma\Lambda}\mathcal{D}_\mu\phi^\Sigma\mathcal{D}^\mu\phi^\Lambda - V(\Phi) + \\
& + \mathcal{L}_{\text{Tensor}} + \mathcal{L}_{\text{Pauli}} + \mathcal{L}_{4-\text{fermions}} + \mathcal{L}_{\text{Others}}\,,
\end{aligned} \tag{B.1}
$$

where the spin connection ω appears in the covariant derivative:

$$\nabla_\mu(\omega) = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\right)$$

and

$$\mathcal{D}\phi^x = D\phi^x + gA^I K_I^x(\phi), \qquad \mathcal{D}q^X = Dq^X + gA^I K_I^X(q)\,.$$

K_I^x and K_I^X are the Killing vectors for the gauging and

$$L_I^{xy} \equiv \partial^y K_I^x, \qquad \omega_{I\,B}{}^A \equiv K_{IX;Y} f_i^{XA} f_B^{Yi}\,,$$

where f_{iA}^X denote the vielbeins to pass the curved indexes u to the flat indexes $iA \in$ USp(2) $\otimes$ USp($2n_H$). $\omega_{X\,i}{}^j(q)$ and $\Delta_{X\,A}{}^B(q)$ are the USp(2) and USp($2n_H$) connections respectively for the quaternionic manifold $\mathcal{Q}$. For what concerns the $\mathcal{S}$ manifold, the completely symmetric constant tensor C_{IJK}, that appears in the Chern-Simons couplings of the vector fields, determines also the geometry of the very special manifold. In fact, $\mathcal{S}$ can be described by an $(n_V + n_T)$–dimensional cubic hypersurface

$$C_{IJK}h^I h^J h^K = 1$$

of an ambient space parametrized by $n_V + n_T + 1$ coordinates $h^I = h^I(\phi^x)$. a_{IJ} are described in terms of h^I and the metric g_{xy}:

$$a_{IJ} = h_I h_J + h_I^x h_J^y g_{xy}\,,$$

where

$$h_{I,x} = \sqrt{\frac{2}{3}}h_{Ix}\,, \qquad h^I{}_{,x} = -\sqrt{\frac{2}{3}}h_x^I\,. \tag{B.2}$$

The Killing vector prepotential $P_{I\,i}{}^j$ depends only on q^u, while $P_{ij} \equiv h^I P_{Iij}$, $P_{ij}^x \equiv h^{xI}P_{Iij}$ and $P_{ij}^{xy} \equiv \delta^{xy}P_{ij} + 4T^{xyz}P_{ij}^z$, where T^{xyz} is a completely symmetric function of ϕ^x, depend on q^u and ϕ^x. From (B.2), one obtains the following useful identity for us:

$$P_{ij}^x = -\sqrt{\frac{3}{2}}P_{ij}^{;x}\,. \tag{B.3}$$

The scalar potential $V(\Phi)$ is given by

$$V = 2g^2 W^x W^x - g_R^2\left[2P_{ij}P^{ij} - P_{ij}^x P^{x\,ij}\right] + 2g^2\mathcal{N}_{iA}\mathcal{N}^{iA}\,. \tag{B.4}$$

The first contribution of (B.4) comes from tensor multiplets, while the rest part comes from vector- and hyper-supermultiplets which under the condition (4.16) gives (4.17). Closure of the SUSY algebra leads:

$$W^x = \frac{\sqrt{6}}{4}h^I K_I^x\,, \qquad \mathcal{N}_{iA} = \frac{\sqrt{6}}{4}f_{XAi}K_I^X h^I\,.$$

In the end, the mass matrices $\mathcal{M}_{Aix}$, $\mathcal{M}_{AB}$ are defined as

$$\mathcal{M}_{Aix} \equiv f_{AiX}K_I^X h^{Ix}, \qquad \mathcal{M}_{AB} \equiv \frac{i\sqrt{6}}{2}f_{AX}^i f_{BiY}K_I^{[Y;X]}h^I\,.$$

More details on the notation and on the derivation of the full action and of the SUSY transformations can be found in [8] and the references therein.

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References

[1] I. Antoniadis, *A possible new dimension at a few TeV*, *Phys. Lett.* **B 246** (1990) 377;
N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315];
N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV* *Phys. Rev.* **D 59** (1999) 086004 [hep-ph/9807344].

[2] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221]; *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[3] E. Cremmer, in *Superspace and supergravity*, S. Hawking and M. Roček eds., Cambridge University Press, (1980);
A.H. Chamseddine and H. Nicolai, *Coupling the* SO(2) *supergravity through dimensional reduction*, *Phys. Lett.* **B 96** (1980) 89.

[4] S.S. Gubser, *Curvature singularities: the good, the bad and the naked*, *Adv. Theor. Math. Phys.* **4** (2002) 679 [hep-th/0002160].

[5] S. Förste, Z. Lalak, S. Lavignac and H.P. Nilles, *A comment on self-tuning and vanishing cosmological constant in the brane world*, *Phys. Lett.* **B 481** (2000) 360 [hep-th/0002164]; *The cosmological constant problem from a brane-world perspective*, *J. High Energy Phys.* **09** (2000) 034 [hep-th/0006139].

[6] C. Csáki, J. Erlich, C. Grojean and T.J. Hollowood, *General properties of the self-tuning domain wall approach to the cosmological constant problem*, *Nucl. Phys.* **B 584** (2000) 359 [hep-th/0004133];
General properties of the self-tuning domain wall approach to the cosmological constant problem, *Nucl. Phys.* **B 584** (2000) 359 [hep-th/0004133].

[7] M. Günaydin and M. Zagermann, *The gauging of five-dimensional, $N = 2$ Maxwell-Einstein supergravity theories coupled to tensor multiplets*, *Nucl. Phys.* **B 572** (2000) 131 [hep-th/9912027]; *The vacua of 5D, $N = 2$ gauged Yang-Mills–Einstein tensor supergravity: abelian case*, *Phys. Rev.* **D 62** (2000) 044028 [hep-th/0002228].

[8] A. Ceresole and G. Dall'Agata, *General matter coupled $N = 2$, $D = 5$ gauged supergravity*, *Nucl. Phys.* **B 585** (2000) 143 [hep-th/0004111].

[9] A. Ceresole, G. Dall'Agata, R. Kallosh and A. Van Proeyen, *Hypermultiplets, domain walls and supersymmetric attractors*, *Phys. Rev.* **D 64** (2001) 104006 [hep-th/0104056].

[10] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, *Modeling the fifth dimension with scalars and gravity*, *Phys. Rev.* **D 62** (2000) 046008 [hep-th/9909134];
K. Skenderis and P.K. Townsend, *Gravitational stability and renormalization-group flow*, *Phys. Lett.* **B 468** (1999) 46 [hep-th/9909070].

[11] R. Altendorfer, J. Bagger and D. Nemeschansky, *Supersymmetric Randall-Sundrum scenario*, *Phys. Rev.* **D 63** (2001) 125025 [hep-th/0003117].

[12] A. Pomarol and T. Gherghetta, *Bulk fields and supersymmetry in a slice of AdS*, hep-th/0003129.

[13] A. Falkowski, Z. Lalak and S. Pokorski, *Supersymmetrizing branes with bulk in five-dimensional supergravity*, *Phys. Lett.* **B 491** (2000) 172 [hep-th/0004093].

[14] E. Bergshoeff, R. Kallosh and A. Van Proeyen, *Supersymmetry in singular spaces*, *J. High Energy Phys.* **10** (2000) 033 [hep-th/0007044].

[15] M. Zucker, *Supersymmetric brane world scenarios from off-shell supergravity*, *Phys. Rev.* **D 64** (2001) 024024 [hep-th/0009083].

[16] A. Falkowski, Z. Lalak and S. Pokorski, *Five-dimensional gauged supergravities with universal hypermultiplet and warped brane worlds*, *Phys. Lett.* **B 509** (2001) 337 [hep-th/0009167].

[17] A. Falkowski, Z. Lalak and S. Pokorski, *Four dimensional supergravities from five dimensional brane worlds*, *Nucl. Phys.* **B 613** (2001) 189 [hep-th/0102145].

[18] J. Bagger, D. Nemeschansky and R.J. Zhang, *Supersymmetric radion in the Randall-Sundrum scenario*, *J. High Energy Phys.* **08** (2001) 057 [hep-th/0012163].

[19] J. Bagger and D.V. Belyaev, *Supersymmetric branes with (almost) arbitrary tensions*, *Phys. Rev.* **D 67** (2003) 025004 [hep-th/0206024].

[20] P. Brax and Z. Lalak, *Brane world supersymmetry, detuning, flipping and orbifolding*, *Acta Phys. Polon.* **B33** (2002) 2399 [hep-th/0207102].

[21] Z. Lalak and R. Matyszkiewicz, *On Scherk-Schwarz mechanism in gauged five-dimensional supergravity and on its relation to bigravity*, *Nucl. Phys.* **B 649** (2003) 389 [hep-th/0210053]; *Twisted supergravity and untwisted super-bigravity*, *Phys. Lett.* **B 562** (2003) 347 [hep-th/0303227].

[22] P. Brax and N. Chatillon, *Detuned branes and supersymmetry breaking*, *J. High Energy Phys.* **12** (2003) 026 [hep-th/0309117].

[23] T. Gherghetta and A. Pomarol, *A warped supersymmetric standard model*, *Nucl. Phys.* **B 602** (2001) 3 [hep-ph/0012378]; *A Stueckelberg formalism for the gravitino from warped extra dimensions*, *Phys. Lett.* **B 536** (2002) 277 [hep-th/0203120].

[24] L.J. Hall, Y. Nomura, T. Okui and S.J. Oliver, *Explicit supersymmetry breaking on boundaries of warped extra dimensions*, *Nucl. Phys.* **B 677** (2004) 87 [hep-th/0302192].

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